



Hilary Term 2010

SEMINAR SERIES

Saïd Business School, University of Oxford



Convenors: Felix Reed-Tsochas, Institute for Science, Innovation and Society, Saïd Business
Eduardo López, Saïd Business School

Our meetings intend to provide a forum for rigorous research (in a broad range of disciplines) focusing on complex adaptive systems, using methods and techniques such as agent-based modelling and complex network analysis. Since potential areas of application for such approaches can be located across the social, natural and engineering sciences, our aim is to involve participants from a wide range of departments in Oxford. We welcome talks which focus on particular areas of application and associated technical issues, but also encourage contributions which address more fundamental conceptual or mathematical problems. The CABDyN Seminar Series is one of the activities of the CABDyN Research Cluster.

Tuesday 16th February, 12:30-14:00

James Martin Seminar Room

Professor Jose Fernando Mendes

Department of Physics, University of Aveiro

“ k -cores in complex networks”

The k -core of a graph is its maximum subgraph in which each node has at least k neighbours within this subgraph. The k -core may be obtained in the following way (the pruning algorithm). Remove from a network all nodes with at least k links. (In graph theory, the number of connections of a node is called its degree.) Some of the remaining vertices may remain with less than k edges. Then remove these vertices, and so on until no further removal is possible. The result, if it exists, is the k -core. Thus, a network is organized as a set of successively enclosed k -cores. The highest k -cores represent most connected and robust areas of a network. Importantly, the notion of the k -core is applicable to a wide range of systems in physics, including disordered lattices. One can see that the k -core is a direct generalization of the percolation cluster (giant connected component in graph theory). Moreover, the k -core problem is among fundamental issues in condensed matter physics and network science [1]. In particular, this problem is directly related to the nature of congestion and jamming phenomena in various systems, to the evolution of magnetization in ferromagnetic materials with random disorder after the magnetic field is applied, etc. Generally, this notion turns out to be relevant in numerous situations, in which the state of a node changes after a given function of the states of the neighbouring nodes exceeds some threshold. The question is: what is the nature of the k -cores in complex networks which are particularly heterogeneous systems? One should also ask: how do these k -cores emerge? We have developed the theory of k -cores in most representative networks having complex architectures [2-4]. These are uncorrelated random graphs with an arbitrary degree distribution [1]. An important feature of these networks is absence of short loops, which makes possible to find an exact solution of the problem. We have applied our theory to complex networks with power-law degree distributions (scale-free networks). The Internet is among these networks. In conclusion, we indicate that the k -core problem is generically related with so-called bootstrap percolation. This is an activation process on a network, in which a node becomes active if at least k of its nearest neighbours are active. Bootstrap percolation is an extensively studied yet rather poorly understood problem in condensed matter physics. Our future work is aimed at the better understanding of relations between these two fundamental problems.

[1] S. N. Dorogovtsev, A. V. Goltsev, and J. F. F. Mendes, *Critical phenomena in complex networks*, *Rev. Mod. Phys.* 80, 1275 (2008).

[2] S. N. Dorogovtsev, A. V. Goltsev, and J. F. F. Mendes, *k -core percolation and k -core organization of complex networks*, *Phys. Rev. Lett.* 96, 040601 (2006).

[3] A. V. Goltsev, S. N. Dorogovtsev, and J. F. F. Mendes, *k -core (bootstrap) percolation on complex networks: Critical phenomena and nonlocal effects*, *Phys. Rev. E* 73, 056101 (2006).

[4] S. N. Dorogovtsev, A. V. Goltsev, and J. F. F. Mendes, *k -core architecture and k -core percolation on complex networks*, *Physica D* 224, 7 (2006).

Please bring your own lunch

For further information contact info.cabdyn@sbs.ox.ac.uk

Seminar webpage: http://sbs-xnet.sbs.ox.ac.uk/complexity/complexity_seminars.asp

Please note: Although the seminar programme detailed above was correct at the time of printing, seminar arrangements are subject to change so, for the latest information please check seminar webpage.